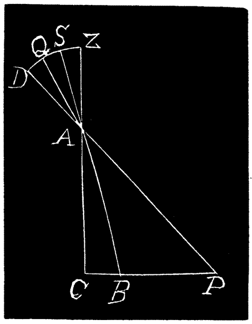
MISCELLANEOUS.

58. Proposed by EDMUND FISH, Hillsbóro, Ill.

The longest noonday winter shadow of an upright object is found to be seven times as long as the shortest summer shadow of the same object. Required the latitude of the place.

I. Solution by S. HART WRIGHT, A. M., M. D., Ph. D., Penn Yan, N. Y.

In the right plane triangles ABC and APC, let the vertical $AC(=\text{unity})$ be the rod that casts a shadow from C to B, and from C to P, when the sun is at S and D. Extend CA to the zenith Z, BA to S and PA to D. Bisect DAS with QA. Let $\angle BAC=\chi=ZAS$. DAS is double the obliquity of the ecliptic$=2\delta=\angle PAB=46^\circ\ 54'\ 30''=v$. $DAQ=SAQ=\delta$, and Q must be on the equator, and QAZ=the required latitude$=\lambda$. $CP=7CB$. Put $7=m$, and $\tan^{-1}m=\beta=81^\circ\ 52'\ 12''$. We have $CB=\tan\chi$, and $CP=\tan(\chi+v)$, and $m\tan\chi=\tan(\chi+v)$, a trigonometric equation, from which we derive $\sin(2\chi\pm v)=\cot(\beta\mp45^\circ)\sin v$. Four values of χ result, the upper signs giving the only acceptable value of $\chi=14^\circ\ 57'\ 30''$. The other signs make the $\angle PAC>90^\circ$. Now $\lambda=\chi+\delta=38^\circ\ 24'\ 45''$ north or south, as the seasons are interchangeable on each side of the equator.



II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let AC be the upright object length$=l$, CP the earth, SB the summer sun, DP the winter sun. Let ϕ=latitude, δ=north, $-\delta$=south declination of the sun, and let the winter shadow be n times as long as the summer shadow.

Then $\angle CAB=(\phi-\delta)$, $\angle CAP=(\phi+\delta)$, $CP=n.CB$, $CP=l\tan(\phi+\delta)$, $CB=l\tan(\phi-\delta)$.

$\therefore\ l\tan(\phi+\delta)=nl\tan(\phi-\delta)$.

$\therefore\ (n+1)\tan^2\phi\tan\delta-(n-1)\tan\phi\sec^2\delta+(n+1)\tan\delta=0$.

$$\tan^2\phi-\frac{2(n-1)\tan\phi}{(n+1)\sin2\delta}+1=0.$$

$$\therefore\ \tan\phi=\frac{(n-2)\pm\sqrt{[(n-1)^2-(n+1)^2\sin^2 2\delta]}}{(n+1)\sin2\delta}\ \ldots\ldots\ldots\ldots\ldots\ldots(A).$$

Now $\delta=23^\circ\ 27'\ 30''$, $n=7$. $\therefore\ \tan\phi=.793428$ or 1.26035.

$\therefore\ \phi=38^\circ\ 25'\ 46''$ or $51^\circ\ 34'\ 14''$.

The two values of $\tan\phi$ are equal when $n=(1+\sin2\delta)/(1-\sin2\delta)$.

$\therefore\ n=6.4118$, $\phi=45^\circ$. When $\phi=\delta$, and $\phi+\delta=90^\circ$, n is infinite.

In the first case the summer shadow is zero and the winter shadow is fi-

nite; in the second case, the winter shadow is infinite and the summer shadow is finite.

In formula (A), δ and n can have any values within proper limits.

Also solved by *W. W. LANDIS*, and *J. SCHEFFER*.

59. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

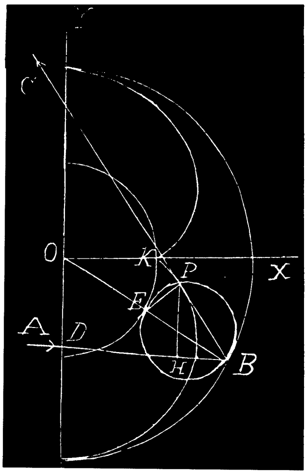
When a cylindrical china jar, standing upon the ground, receives the sun's rays obliquely, a bright curve is observed to form itself at the bottom of the jar, and it is found that the shape and dimensions of this curve are not affected by the varying elevations of the sun: account for this latter circumstance, and determine the nature of the bright curve. [From *Parkinson's Optics*.]

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Mississsppi.

All rays striking any element of the cylindrical surface lie in a vertical plane. Their reflections form the other face of the dihedral angle whose bisector passes through the axis of the cylinder. These reflected rays intersect the base of the cylinder in a straight line. There is thus formed a system of lines, and the bright curve observed is their envelope. The altitude of the sun does not affect the position of the vertical planes; and, therefore, the intersections with the bottom of the jar are unchanged, and the continual intersection of the consecutive lines so formed produces a curve invariable as to its shape and size.

The bright curve is the caustic by reflection for the circle, the incident rays being parallel. The following general property of caustics by reflection for parallel rays is established in *Price's Infinitesimal Calculus*: "The distance from the incident point in the reflecting curve to the point of intersection of two consecutive reflected rays, is equal to one-fourth of the chord of the circle of curvature at the point of incidence which is parallel to the incident ray."

A. Take the center of the circle as the origin, the X-axis parallel to the incident rays, the Y-axis perpendicular to them. Let AB be an incident ray, BC its reflection, the angle between them being 2θ. Take BP along BC equal to one-half of DB, D being the intersection of AB with the Y-axis. Then, according to the principle quoted above, P is a point of the caustic. To find the locus of P: Draw OB, denoting it by a. From P drop a perpendicular to AB meeting it at H. Denoting the coördinates of P by x and y,



$$x=DB-HB=a\cos\theta-\tfrac{1}{2}a\cos\theta\cos2\theta,$$

$$y=OD-PH=a\sin\theta-\tfrac{1}{2}a\cos\theta\sin2\theta.$$

From these $x=\tfrac{3}{4}a\cos\theta-\tfrac{1}{4}a\cos3\theta$,

and $y=\tfrac{3}{4}a\sin\theta-\tfrac{1}{4}a\sin3\theta$.

These may be written